NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

REPORT NO.
TDR-594(1114)TN-2

Optimum Propellant Loading and Propellant Utilization System Techniques

(A Tutorial Report)

1 MAY 1961

Prepared by DAVID W. WHITCOMBE

Prepared for DEPUTY COMMANDER AEROSPACE SYSTEMS
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
Inglewood, California

AEROSPACE CORPORATION
CONTRACT NO. AF 04(647)-594

Report No.
TDR-594(1114)TN-2

OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES

(A Tutorial Report)

by

David W. Whitcombe

AEROSPACE CORPORATION
El Segundo, California

Contract No. AF 04(647)-594

1 May 1961

Prepared for

DEPUTY COMMANDER AEROSPACE SYSTEMS
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
Inglewood, California

PREFACE

A similar report was issued 30 December 1957 at Space Technology Laboratories, Inc., "Optimum Propellant Loading for Titan and Thor Missiles" (Confidential), by David W. Whitcombe. In the present unclassified edition, Thor and Titan performance data are excluded. Sections have been added describing the calculation of mean outage and the operation of a propellant utili ation system.

ABSTRACT

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $\langle\theta^2\rangle$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta < \theta_0)$.

The analysis in this report applies directly to conventionally loaded ballistic missiles. It is assumed that the missile stage is loaded in accordance with a loading mixture ratio, r_L. That is, the propellant is loaded such that the ratio of oxidizer to fuel is r_L in the nominal missile. A mixture ratio bias, Δr, is calculated that will minimize θ_N and $\langle\theta^2\rangle$, and maximize $P(\theta < \theta_0)$. An identification is obtained that extends the basic loading formulae to stages employing PU systems. Additional formulae are given that allow calculation of a fuel or oxidizer bias equivalent to the mixture ratio bias.

CONTENTS

SECTION 1

INTRODUCTION AND SUMMARY

It is shown in this report that the total propellant per stage in the Titan and Thor missiles is best loaded with the aid of a loading mixture ratio, r_L. When r_L is obtained, the nominal initial masses of oxidizer and fuel, M_{LN} and M_{FN}, respectively, to be loaded are given by the relationships:

$$M_{FN} = \frac{M_N}{1 + r_L} \tag*{*(3.2a)}$$

$$M_{LN} = \frac{M_N r_L}{1 + r_L} \tag{3.2b}$$

where M_N is the total initial mass of propellant in the nominal missile. The optimum value for r_L is less than the nominal expected burning mixture ratio, r_N; that is,

$$r_L = r_N - \Delta r \tag{3.5}$$

Formulae for the computation of Δr will be derived in this report.

Loading the Titan or Thor in accordance with r_L is equivalent to loading the missile in accordance with r_N and then adding an excess of fuel, ΔM_F, or, equivalently removing a portion of the oxidizer, ΔM_L. The quantities ΔM_F

---

*Equation numbers in this section refer directly to equations derived in Sections 2 through 8.

and ΔM_L are related to Δr by the following relationship:

$$\frac{\Delta M_F}{M_{FN}} = \frac{\Delta r}{r_N} = \frac{-\Delta M_L}{M_{LN}} \tag{3.7}$$

The quantity ΔM_F is referred to as the "fuel bias."

An attempt is made to load the Titan and Thor to the desired loading mixture ratio. The propellant is loaded using weight-measuring load cells or using volume-measuring flow devices. In either case, errors in quantity of both oxidizer and fuel will exist. In addition, the actual burning mixture ratio will differ from the intended burning mixture ratio. As a result of these errors, an excess of either oxidizer or fuel will remain in the tanks when the other is exhausted. The residual unburned propellant is referred to as <u>outage</u>. Outage will be denoted by the symbol θ, and will be always be expressed as a fraction of the total stage propellant.

A propellant utilization (PU) control system may be developed that will cause the propellant to be burned in closed-loop fashion. Such a technique is employed in the Atlas missile. This method requires measurement during flight of both the remaining oxidizer and the remaining fuel. The burning mixture ratio is then adjusted so that the propellants are exhausted simultaneously. Errors in the measurement of remaining oxidizer and fuel result in outage even when a PU system is used. The outage can be minimized by introducing a known bias in the fuel measurement. It will be shown in this report that choosing the fuel bias for a PU system is analogous to choosing the loading mixture ratio for the Titan and Thor missiles.

The following statistical properties of outage are derived in the body of this report:

$\langle\theta^2\rangle$ = mean square outage

θ_N = mean outage

$P(\theta < \theta_0)$ = probability that the outage is less than some fixed constant (θ_0)

$$\sigma_\theta^2 = \text{outage variance}$$

$$= \langle\theta^2\rangle - \theta_N^2$$

$$\sigma_\theta = \sqrt{\sigma_\theta^2}$$

Five particular methods of loading a missile that does not have a PU system are discussed:

1. Standard loading: In this method $r_L = r_N$, the nominal expected burning mixture ratio. This procedure is discussed in Section 2.

2. Minimum mean square outage loading: In this method $r_L = r_N - \Delta r$, where Δr is chosen to minimize the root mean square (rms) outage. The method for determining Δr is given in Section 3.

3. Maximum $P(\theta < \theta_0)$ loading: In this method, $r_L = r_N - \Delta r$, where Δr is chosen to maximize $P(\theta < \theta_0)$. This method is discussed in Section 4.

4. Minimum mean outage loading: In this method $r_L = r_N - \Delta r$, where Δr is chosen to minimize the mean outage. This procedure, discussed in Section 5, is used when it is desired to maximize the performance of the nominal missile.

5. Maximum $P(V > V_0)$ loading: In this method, discussed in Section 6, Δr is chosen to maximize $P(V > V_0)$, where V_0 is a general performance parameter.

An analog for each of the above loading procedures exists when a PU system is employed. The only change is that a fuel measurement bias, $\Delta f (\equiv \Delta M_F)$, (rather than a mixture ratio bias) is obtained to achieve the stated condition.

An example, using the formulae developed in this report, will be given. It will be assumed in the example that the missile does not have a PU system. The following data on the missile will be used:

$$M_{FN} = 60,000 \text{ lb}$$

$$M_{LN} = 135,000 \text{ lb}$$

$$M_N = 195,000 \text{ lb}$$

$$r_N = 2.25$$

The following data on propellant loading and burning will be assumed:

Initial fuel mass loading tolerance $(\sigma_F) = 0.25\%$

Initial oxidizer mass loading tolerance $(\sigma_L) = 0.5\%$

Burning mixture ratio tolerance $(\sigma_r) = 0.5\%$

In Table I, the quantities $\theta_N, \sqrt{<\theta^2>}$, and σ_θ are given as fractions of the total nominal mass of propellant (195,000 lb). Note that when the mean outage is minimized, the rms outage is near minimum, and that the $P(\theta < 1\%)$ is near maximum. For most missions, acceptable results will be obtained when θ_N is minimized.

A similar table could readily be constructed for a missile stage utilizing a PU system. It is shown in Section 8 that the principal change in the analysis results from redefining σ and σ_L and choosing r

Summary of Equations

The more important relationships and definitions derived in the body of the report will be summarized in the following paragraphs. The equations will be given assuming conventional loading procedures as employed in the Titan and Thor stages. The following definitions are used:

M_N = nominal total mass of propellant loaded

M_{FN} = nominal total mass of fuel loaded

$$= \frac{M_N}{1 + r_L} \tag{3.2a}$$

M_{LN} = nominal total mass of oxidizer loaded

$$= \frac{M_N r_L}{1 + r_L} \tag{3.2b}$$

r_N = nominal expected burning mixture ratio

r_L = loading mixture ratio $= r_N \cdot \Delta r$ (3.5)

σ_F = initial fuel mass loading tolerances (1σ), expressed as a fraction M_{FN}

σ_L = initial oxidizer mass loading tolerance (1σ), expressed as a fraction of M_{LN}

σ_r = burning mixture ratio tolerance (1σ), expressed as a fraction of r_N

$$p = \sqrt{\sigma_F^2 + \sigma_L^2 + \sigma_r^2} \tag{2.15}$$

$$a = \frac{\Delta r}{p\, r_N} \tag{3.12}$$

$$\phi(a) = \frac{1}{\sqrt{2\pi}} \int_0^{a} e^{-\frac{1}{2}u^2}\, du$$

$$\phi'(a) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}a^2}$$

$\bar{\theta}_N$ = mean outage

$\langle \theta^2 \rangle$ = mean square outage

σ_θ^2 = outage variance (about the mean)

α_0 = some fixed constant value of outage

$P(\theta < \theta_0)$ = probability that the outage is less than θ_0

The quantities θ_N, $\sqrt{\langle\theta^2\rangle}$, σ_θ, and θ_0 are all measured as fractions of the total mass of propellant.

Mean outage:

The expression for mean outage is calculated in the body of the report as

$$\theta_N = p \left[\phi'(\alpha) + \alpha \phi(\alpha) - \frac{\alpha}{2} \frac{r_N - 1}{r_N + 1} \right] \tag{5.5}$$

The mean outage is minimized when

$$\varphi(\alpha) = \frac{1}{2} \frac{r_N - 1}{r_N + 1} \tag{5.11}$$

Then

$$\theta_N = p\phi'(\alpha) \tag{5.12}$$

$$\langle\theta^2\rangle = p^2 \left[\frac{(1 + \alpha^2) r_N}{(1 + r_N)^2} - \frac{r_N - 1}{r_N + 1} \alpha \phi'(\alpha) \right] \tag{5.17}$$

Table III may be used when the propellants are loaded to minimize the mean outage. Several statistical parameters are calculated as a function of mixture ratio, r_N.

Mean square outage:

The mean square outage is obtained as:

$$<\theta^2> = \frac{p^2(r_N^2 + 1)}{2(1 + r_N)^2}(1 + a^2)$$

$$- \frac{r_N - 1}{r_N + 1} p^2\left[a\phi'(a) + (1 + a^2)\,\phi(a)\right] \qquad (3.16, 5.16)$$

The mean square outage is minimized when

$$\phi'(a) + a\phi(a) = \frac{r_N^2 + 1}{2(r_N^2 - 1)} a \qquad (3.20)$$

Then

$$\theta_N = \frac{pa\, r_N}{r_N^2 - 1}$$

$$<\theta^2> = p^2\left[\frac{1}{2} - \frac{r_N^2 - 1}{(1 + r_N)^2}\phi(a)\right]$$

The outage variance may be determined in general from Equations (3.16) and (5.5) as

$$\sigma_\theta^2 = <\theta^2> - \theta_N^2 \qquad (5.13)$$

Probability that $\theta < \theta_0$:

$$P(\theta < \theta_0) = \phi(u_0) + \phi(u_1) \tag{4.11a}$$

where

$$u_0 = \frac{(1 + r_N)\theta_0}{p\, r_N} + \alpha \tag{4.11b}$$

$$u_1 = \frac{(1 + r_N)\theta_0}{p} - \alpha \tag{4.11c}$$

The quantity $P(\theta < \theta_0)$ is maximized when

$$\alpha = \frac{r_N^2 - 1}{2p\, r_N}\, \theta_0 \tag{4.14}$$

In this special case, Equations (4.11) become

$$P(\theta < \theta_0) = 2\phi(u_0) \tag{4.16}$$

where

$$u_0 = \frac{(1 + r_N)^2}{2p\, r_N}\, \theta_0 \tag{4.15}$$

Table IV may be used when the propellants are loaded to maximize $P(\theta < \theta_0)$. When this table is used, it is assumed that θ_0/p is obtained by substituting $u_0 = 3$ into Equation (4.15). Several statistical parameters of interest are calculated in this table.

Probability that θ = fuel or oxidizer:

The probability that the outage will be all fuel or all oxidizer is dependent on α. It is shown in the text that these probabilities are evaluated as:

$$P(\theta = \text{fuel}) = \frac{1}{2} + \phi(\alpha)$$

$$P(\theta = \text{oxidizer}) = \frac{1}{2} - \phi(\alpha) \tag{5.21}$$

PU System Analysis

The following definitions are used in the analysis of PU systems:

σ_f = rms fuel measurement error (1σ), expressed as a fraction of M_{FN}

σ_ℓ = rms oxidizer measurement error (1σ), expressed as a fraction of M_{LN}

$q = \sqrt{\sigma_f^2 + \sigma_\ell^2}$

Δf = fuel measurement bias in pounds (to be subtracted from each PU system fuel measurement)

All outage statistical parameters that result when a PU system is employed may be obtained using the relations that apply for conventional loading procedures. It is only necessary to make the following identification:

$$p \equiv q \tag{8.11a}$$

$$\alpha = \frac{\Delta f}{q M_{FN}} \tag{8.11b}$$

Note that the PU system fuel bias Δf is equivalent to the fuel loading bias discussed in Equation (3. 7a).

The quantities $\phi(\alpha)$ and $\phi'(\alpha)$ are tabulated in Mathematical Tables from Handbook of Chemistry and Physics, (Chemical Rubber Publishing Co., Cleveland, Ohio). For convenience, an abridged tabulation of these functions is given in Table II.

Table I. Comparison of four loading procedures for the hypothetical missile.

Loading procedure	Mean outage	RMS outage		Outage standard deviation (about the mean)		Loading mixture ratio	Fuel bias (excess fuel)	Oxidizer deficiency
	θ_N	$\sqrt{\langle \theta^2 \rangle}$	$P(\theta < 1\%)$	σ_θ	$\Delta r = r_N - r_L$	r_L	ΔM_F	$-\Delta M_L$
	(%)	(%)		(%)			(lb)	(lb)
Standard	0.2992	0.4018	0.9730	0.2682	0	2.2500	0	0
Minimum mean square outage	0.2655	0.3312	0.9948	0.1980	0.0108	2.2392	288	649
Maximum $P(\theta < 1\%)$	0.3195	0.3751	0.9982	0.1965	0.0203	2.2297	542	1219
Minimum mean outage	0.2638	0.3344	0.9924	0.2055	0.0085	2.2415	226	609

Table II. Tabulation of functions.

a	$\phi(a)$	$\phi'(a)$	$\phi'(a) + a\phi(a)$
0	0	0.3989	0.39890
0.2	0.0793	0.3910	0.40686
0.4	0.1554	0.3683	0.43046
0.6	0.2258	0.3332	0.46868
0.8	0.2881	0.2897	0.52018
1.0	0.3413	0.2420	0.58330
1.2	0.3849	0.1942	0.65608
1.4	0.4192	0.1497	0.73658
1.6	0.4452	0.1109	0.82322
1.8	0.4641	0.0790	0.91438
2.0	0.4773	0.0540	1.00860
2.2	0.4861	0.0355	1.10492
2.4	0.4918	0.0224	1.20272
2.6	0.4953	0.0136	1.30138
2.8	0.4974	0.0079	1.40063

Table III. Calculation of outage parameters as a function of nominal burning mixture ratio when the missile stage is loaded to minimize mean outage, θ_N.

r_N	$\phi(a)$	a	$\frac{\theta_N}{P} = \phi'(a)$	$\frac{\langle\theta^2\rangle}{P^2}$	$\frac{\sigma_\theta^2}{P^2}$	$\frac{\sigma_\theta}{P}$	$\frac{\theta_0}{P}$ when $P(\theta < \theta_0) = 0.9974$
1.0	0	0	0.3989	0.2500	0.09198	0.3033	1.50
1.25	0.0556	0.1398	0.3951	0.2456	0.0895	0.2992	1.52
1.5	0.1000	0.2533	0.3864	0.2358	0.0865	0.2941	1.54
1.75	0.13636	0.3489	0.3754	0.2239	0.0830	0.2881	1.56
2.0	0.1667	0.4308	0.3636	0.2113	0.0791	0.2812	1.58
2.25	0.1923	0.5023	0.3517	0.1988	0.0751	0.2740	1.59
2.5	0.2143	0.5659	0.3399	0.1870	0.0715	0.2674	1.60
3.0	0.2500	0.6745	0.3176	0.1657	0.0648	0.2546	1.59
4.0	0.3000	0.7418	0.2799	0.1320	0.0537	0.2317	1.57
5.0	0.3333	0.9672	0.2499	0.1077	0.0452	0.2126	1.53
6.0	0.3571	1.0674	0.2257	0.0899	0.0390	0.1975	1.49

Table IV. Outage statistical parameters when $P(\theta < \theta_0)$ is maximized; $\frac{\theta_0}{p}$ is chosen such that $P(\theta < \theta_0) = 0.9974$.

r_N	$\alpha = 3\frac{r_N - 1}{r_N + 1}$	$\frac{\theta_0}{p} = \frac{6 r_N}{(1 + r_N)^2}$	$\phi(\alpha)$	$\phi'(\alpha)$	$\frac{\theta_N}{p}$	$\frac{\langle\theta^2\rangle}{p^2}$	$\frac{\sigma_\theta}{p}$
1.0	0	.5000	0.0000	0.3989	0.3989	0.2500	0.3015
1.25	0.3333	1.4815	0.1306	0.3774	0.4024	0.2511	0.2987
1.50	0.6000	.4400	0.2258	0.3332	0.4087	0.2522	0.2919
1.75	0.8182	1.3884	0.2933	0.2855	0.4139	0.2512	0.2827
2.0	1.0000	1.3333	0.3413	0.2420	0.4166	0.2474	0.2717
2.25	1.1538	1.2781	0.3757	0.2050	0.4166	0.2413	0.2602
2.5	1.2857	1.2245	0.4007	0.1745	0.4142	0.2332	0.2482
3.0	1.5000	1.1250	0.4332	0.1295	0.4043	0.2145	0.2258
4.0	1.8000	0.9600	0.4641	0.0790	0.3744	0.1756	0.1881
5.0	2.0000	0.8333	0.4773	0.0540	0.3419	0.1425	0.1600
6.0	2.1429	0.7347	0.4839	0.0402	0.3117	0.1171	0.1411

SECTION 2

OUTAGE (WITH STANDARD LOADING)

In this section, an expression for the mean square outage for a single stage is derived; the calculation involves these parameters:

M = initial mass of propellant

M_F = initial mass of fuel

M_L = initial mass of oxidizer

r = mixture ratio.

Mass flow rates are indicated as $\dot{M}$, $\dot{M}_F$, and $\dot{M}_L$; a subscript N is used to indicate nominal values. The burning mixture ratio is defined as

$$r = \frac{\dot{M}_L}{\dot{M}_F} \quad . \tag{2.1}$$

Hence,

$$\dot{M}_F = \frac{\dot{M}}{1 + r} \text{ and } \dot{M}_L = \frac{r\dot{M}}{1 + r} \quad . \tag{2.2}$$

All the quantities defined above are assumed to be constants in the following discussion.

The missile engine will operate until the supply of either fuel or oxidizer is exhausted. The remaining amount of fuel or oxidizer is called outage. Note that outage is always positive and is always <u>all</u> fuel or <u>all</u> oxidizer. The outage calculation then depends on whether the outage is all fuel or all oxidizer.

When the outage is all fuel, the burning time, t_L, is given by

$$t_L = \frac{M_L}{\dot{M}_L} \quad . \tag{2.3}$$

The mass of fuel consumed during this period of time is then

$$\Delta M_F = \dot{M}_F t_L = \dot{M}_F \frac{M_L}{\dot{M}_L} \quad . \tag{2.4}$$

The fuel outage may then be written as

$$\theta_F = M_F - \dot{M}_F \frac{M_L}{\dot{M}_L} \quad . \tag{2.5}$$

Similarly, if the fuel burns out first, the outage, all oxidizer, can be calculated as

$$\theta_L = M_L - \dot{M}_L \frac{M_F}{\dot{M}_F} \quad . \tag{2.6}$$

The above expressions may be written as

$$\theta_F = M_F - \frac{1}{r} M_L \tag{2.7a}$$

$$\theta_L = M_L - r M_F \quad . \tag{2.7b}$$

Note that the above two events are mutually exclusive because the outage is always positive. The outage is calculated using Equation (2.7a) or Equation (2.7b) depending on which gives a positive result.

There can be no outage for the nominal missile in the standard method of propellant loading; that is, an attempt is made to initially load the missile such that

$$M_{FN} = \frac{M_N}{1 + r_N} \text{ and } M_{LN} = \frac{r_N M_N}{1 + r_N} \quad . \tag{2.8}$$

If the missile were actually loaded this way and the mixture ratio were nominal, there would never be any outage in a nominal missile. However, because of ground and inflight temperature variations, propellant measurement tolerances, and burning mixture ratio tolerances, this performance can never be achieved.

The linear variation in outage may be obtained as a function of the tolerances on M_F, M_L, and r. This result is:

$$\delta\theta_F = \delta M_F + \frac{\delta r}{r_N^2} M_{LN} - \frac{1}{r_N} \delta M_L \tag{2.9a}$$

or

$$\delta\theta_L = \delta M_L - \delta r \, M_{FN} - r_N \, \delta M_F \quad . \tag{2.9b}$$

The mean square outage may now be calculated using Equation (2.9a) or (2.9b) whichever gives a positive result. Assume now that the loading is accomplished according to Equation (2.8) and δM_F, δr, and δM_L are uncorrelated and normally distributed with zero mean. Then the outage is equally likely to be all fuel or all oxidizer. The mean square outage, $<\theta^2>$, written as a fraction of the total propellant, may then be evaluated as

$$<\theta^2> = \frac{X_F^2 + X_L^2}{2} \tag{2.10}$$

where X_F^2 and X_L^2 are mean square fuel and oxidizer outages expressed as fractions of the total propellant. These quantities may be calculated from Equation (2.9) by squaring, averaging, and dividing by the square of the nominal mass of propellant as

$$X_F^2 = \frac{\left\langle (\delta\theta_F)^2 \right\rangle}{M_N^2}$$

$$= \frac{1}{(1 + r_N)^2} (\sigma_F^2 + \sigma_L^2 + \sigma_r^2) \tag{2.11}$$

and

$$X_L^2 = \frac{\left\langle (\delta\theta_L)^2 \right\rangle}{M_N^2}$$

$$= \frac{r_N^2}{(1 + r_N)^2} (\sigma_F^2 + \sigma_L^2 + \sigma_r^2) \tag{2.12}$$

where

$$\sigma_F^2 = \left\langle \left(\frac{\delta M_F}{M_{FN}} \right)^2 \right\rangle$$

$$\sigma_L^2 = \left\langle \left(\frac{\delta M_L}{M_{LN}} \right)^2 \right\rangle$$

$$\sigma_r^2 = \left\langle \left(\frac{\delta r}{r_N} \right)^2 \right\rangle \tag{2. 13}$$

and $\langle \cdots \rangle$ indicates an ensemble average. It is assumed in the above equations that the missile is loaded in accordance with Equation (2. 8).

When the above result is substituted into Equation (2. 10), the following expression for mean square outage is obtained.

$$\langle \theta^2 \rangle = \frac{1}{2} \frac{r_N^2 + 1}{(r_N + 1)^2} p^2 \tag{2. 14}$$

where

$p^2 = \sigma_F^2 + \sigma_L^2 + \sigma_r^2$

σ_F = rms fuel tolerance (1σ), expressed as a fraction of M_{FN}

σ_L = rms oxidizer tolerance (1σ), expressed as a fraction of M_{LN}

σ_r = rms burning mixture ratio tolerance (1σ), expressed as a fraction of r_N. (2. 15)

SECTION 3

MINIMUM MEAN SQUARE OUTAGE LOADING

It is clear from Equation (2.7) that the expected oxidizer outage is r_N times the expected fuel outage when the missile is loaded in the standard way [i.e., in accordance with Equation (2.8)]. Thus, an attempt should be made to prevent the oxidizer from burning out first. This may be accomplished by loading more fuel and less oxidizer than standard, keeping the same total mass of propellant. This is equivalent to loading the missile on the basis of a loading mixture ratio, r_L, where

$$r_L \leq r_N \tag{3.1}$$

Then the missile would be loaded such that

$$M_{FN} = \frac{M_N}{1 + r_L} \tag{3.2a}$$

$$M_{LN} = \frac{M_N r_L}{1 + r_L} \quad . \tag{3.2b}$$

The result of such loading is a surplus of fuel in the nominal missile, after all the oxidizer is burned. Assume now that the missile is loaded according to Equation (3.2). Then, using similar methods as were used in Section 2, the linearized fuel and oxidizer outages may be written as

$$\delta\sigma_F = \delta M_F + \frac{\delta r}{r_N} M_{LN} - \frac{1}{r_N} \delta M_L + \frac{\Delta r}{r_N} M_{FN} \tag{3.3}$$

and

$$\delta\theta_L = -r_N\left(\delta M_F + \frac{\delta r}{r_N} M_{FN} - \frac{\delta M_L}{r_N} + \frac{\Delta r}{r_N} M_{FN}\right) \tag{3.4}$$

where

$$\Delta r = r_N - r_L \quad . \tag{3.5}$$

To the first approximation

$$\frac{M_{LN}}{r_N} \approx M_{FN}$$

hence, the above equations may be written as

$$\frac{\delta\theta_F}{M_N} = \frac{M_{FN}}{M_N}\left(\frac{\delta M_F}{M_{FN}} + \frac{\delta r}{r_N} - \frac{\delta M_L}{M_{LN}} + \frac{\Delta r}{r_N}\right) \tag{3.6a}$$

and

$$\frac{\delta\theta_L}{M_N} = \frac{-r_N M_{FN}}{M_N}\left(\frac{\delta M_F}{M_{FN}} + \frac{\delta r}{r_N} - \frac{\delta M_L}{M_{LN}} + \frac{\Delta r}{r_N}\right) \quad . \tag{3.6b}$$

Note that Equation (3.6) would be equally valid if the substitutions

$$\frac{\Delta r}{r_N} = \frac{\Delta M_F}{M_{FN}} \qquad (3.7a)$$

or

$$\frac{\Delta r}{r_N} = \frac{-\Delta M_L}{M_{LN}} \qquad (3.7b)$$

were made in Equations (3.6). When Equation (3.7a) is used, the fuel bias, ΔM_F, rather than the mixture ratio bias, Δr, would be obtained. The calculation of the fuel bias is convenient when it is desired to always load the oxidizer tank to capacity. The fuel tank is loaded with the nominal quantity of fuel plus the fuel bias. Equation (3.7b) calculates an oxidizer bias, ΔM_L. This quantity would be useful if the fuel tank were always loaded to capacity. The analysis used in this report will be presented in terms of the mixture ratio bias. The fuel bias, or the oxidizer bias can always be calculated, if desired, using Equations (3.7).

Outage may now be calculated using Equations (3.6a) or (3.6b), whichever gives a positive result. Equations (3.6) may be written in the following way,

$$\frac{\delta\theta}{M_N} = \frac{1}{1 + r_N} X, \qquad X \geq 0$$

$$= \frac{-r_N}{1 + r_N} X, \qquad X \geq 0 \qquad (3.8)$$

where

$$X = \frac{\delta M_F}{M_{FN}} + \frac{\delta r}{r_N} - \frac{\delta M_L}{M_{LN}} + \frac{\Delta r}{r_N}$$

The quantity X is now normally distributed with mean $\frac{\Delta r}{r_N}$. Then the mean square outage, $< \theta^2 >$, may be written as

$$< \theta^2 > = \frac{1}{(1 + r_N)^2} \int_0^{\infty} x^2 F(x)\, dx + \frac{r_N^2}{(1 + r_N)^2} \int_{-\infty}^{0} x^2 F(x)\, dx \qquad (3.9)$$

where

$$F(x) = \frac{1}{\sqrt{2\pi}\, p}\, e^{-\frac{1}{2}p^2 \left(x - \frac{\Delta r}{r_N}\right)^2} \qquad (3.10)$$

and, as defined in Equation (2. 15),

$$p^2 = \sigma_F^2 + \sigma_L^2 + \sigma_r^2$$

Equation (3. 10) may now be substituted into Equation (3. 9). After a change in variables, the result is

$$<\theta^2> = \frac{1}{(1 + r_N)^2} \int_{-a}^{\infty} \left(tp + \frac{\Delta r}{r_N} \right)^2 \phi'(t)\, dt$$

$$+ \frac{r_N^2}{(1 + r_N)^2} \int_{a}^{\infty} \left(tp - \frac{\Delta r}{r_N} \right)^2 \phi'(t)\, dt \tag{3.11}$$

where

$$\phi'(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} t^2}$$

$$a = \frac{\Delta r}{p r_N} \quad . \tag{3.12}$$

Expanding the parentheses in Equation (3. 11) and simplifying gives the result

$$<\theta^2> \frac{(1 + r_N)^2}{p^2} = \int_{-a}^{\infty} t^2 \phi'(t)\, dt + 2a \int_{-a}^{\infty} t \phi'(t)\, dt + a^2 \int_{-a}^{\infty} \phi'(t)\, dt$$

$$+ r_N^2 \int_{a}^{\infty} t^2 \phi'(t)\, dt - 2a\, r_N^2 \int_{a}^{\infty} t \phi'(t)\, dt$$

$$+ a^2 r_N^2 \int_{a}^{\infty} \phi'(t)\, dt \quad . \tag{3.13}$$

The above result may be simplified with the aid of the following identities.

$$\frac{1}{\sqrt{2\pi}} \int_x^{\infty} t^2 \phi'(t)\, dt = \frac{1}{2} + x\phi'(x) - \phi(x) \tag{3.14a}$$

$$\frac{1}{\sqrt{2\pi}} \int_x^{\infty} t e \phi'(t)\, dt = \phi'(x) = \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} \tag{3.14b}$$

$$\frac{1}{\sqrt{2\pi}} \int_0^{x} e^{-\frac{1}{2}t^2}\, dt = \phi(x) \quad . \tag{3.14c}$$

The resultant simplification is the following.

$$\begin{aligned} <\theta^2> \frac{(1 + r_N)^2}{p^2} = {} & \frac{1}{2} - a\phi'(a) + \phi(a) + 2a\phi'(a) + a^2 \left[\frac{1}{2} + \phi(a)\right] \\ & + r_N^2 \left\{ \frac{1}{2} + a\phi'(a) - \phi(a) - 2a\phi'(a) \right. \\ & \left. + \left[a^2 \; \frac{1}{2} - \phi(a)\right] \right\} \end{aligned} \tag{3.15}$$

The above result may be simplified to obtain the mean square outage in the following form.

$$<\theta^2> = p^2 \frac{r_N^2 + 1}{2(1 + r_N)^2} (1 + a^2) - \frac{r_N - 1}{r_N + 1} p^2 \left[a\phi'(a) + (1 + a^2)\, \phi(a)\right] \tag{3.16}$$

where α, $\phi(\alpha)$, and $\phi'(\alpha)$ are defined in Equations (3.12) and 3.14).*

Now $\alpha = \dfrac{\Delta r}{p r_N}$ may be chosen to minimize $<\theta^2>$; that is, choose α to satisfy the equation

$$\frac{\partial <\theta^2>}{\partial \alpha} = 0 \qquad (3.17)$$

then

$$(r_N^2 + 1)\,\alpha = (r_N^2 - 1)\left[\alpha\,\phi''(\alpha) + (2 + \alpha^2)\,\phi'(\alpha) + 2\alpha\,\phi(\alpha)\right] .$$

This equation may be simplified by noting that

$$\phi''(\alpha) = -\,\alpha\,\phi'(\alpha) \quad . \qquad (3.19)$$

Then the condition that the mean square outage is a minimum is expressed by the following equation:

$$\phi'(\alpha) + \alpha\,\phi(\alpha) = \frac{r_N^2 + 1}{2(r_N^2 - 1)}\,\alpha \quad . \qquad (3.20)$$

Tables giving the normal error function $\phi(\alpha)$ as well as the derivative $\phi'(\alpha)$ are given in the Mathematical Tables from the Handbook of Chemistry and Physics.

---

*The above method and statistical calculations were furnished by R.A. Moore.

The solution to Equation (3.20) gives α as a function of r_N. When $\alpha = \frac{\Delta r}{p r_N}$ is known, the quantity Δr may be evaluated. Then the loading mixture ratio, r_L, may be evaluated as

$$r_L = r_N - \Delta r$$

$$= r_N(1 - \alpha p) \quad . \tag{3.21}$$

Minimum mean square outage loading results when the missile stage is loaded on the basis of r_L, in accordance with Equations (3.2), (3.20), and (3.21).

Table V gives a tabulation of Equation (3.16) for the case when $r_N = 2.25$. Note that minimum mean square outage occurs when $\alpha \approx 0.6$. The minimum value could also be obtained from Equation (3.20).

A comparison of standard loading and minimum mean square outage loading, when $r_N = 2.25$, can be made using Table V Standard loading requires that the missile be loaded with $\alpha = 0$. With minimum mean square outage loading, $\alpha = 0.6$ is used. The comparison will be made assuming $\sigma_L = \sigma_r = 0.5\%$, $\sigma_F = 0.25\%$. Then $p = 0.75\%$. When standard loading is used,

$$\sqrt{<\theta^2>} = 0.536p$$

$$= 0.402\% \quad . \tag{3.22}$$

When minimum $<\theta^2>$ loading is used,

$$\sqrt{<\theta^2>} = 0.441p$$

$$= 0.331\% \tag{3.23}$$

SECTION 4

PROBABILITY THAT THE OUTAGE IS LESS THAN A GIVEN VALUE

When the guaranteed range of a missile stage is calculated, it is assumed that an excess of propellant is on board to allow for the existence of outage. The amount of additional propellant depends on the probability that the actual outage, θ, will be less than some reference outage, denoted by θ_0. Recall that both θ and θ_0 are measured as percentages of the total propellant loaded per stage.

It is shown in Equation (2.7) that the fuel and oxidizer outage, expressed as a fraction of total stage propellant, is given by

$$\frac{\delta\theta_F}{M_N} = \frac{1}{1 + r_N}\left(\frac{\delta M_F}{M_{FN}} + \frac{\delta r}{r_N} - \frac{\delta M_L}{M_{LN}} + \frac{\Delta r}{r_N}\right) \tag{4.1}$$

and

$$\frac{\delta\theta_L}{M_N} = \frac{-r_N}{1 + r_N}\left(\frac{\delta M_F}{M_{FN}} + \frac{\delta r}{r_N} - \frac{\delta M_L}{M_{LN}} + \frac{\Delta r}{r_N}\right) . \tag{4.2}$$

That is, $\frac{\delta\theta_F}{M_N}$ and $\frac{\delta\theta_L}{M_N}$ are perfectly correlated, since

$$\frac{\delta\theta_L}{M_N} = -r_N \frac{\delta\theta_F}{M_N} . \tag{4.3}$$

The actual outage, θ, may then be written in terms of the random variable, X, as

$$\theta = \frac{X}{1 + r_N} \quad \text{if } X \geq \theta$$

$$\theta = \frac{- X r_N}{1 + r_N} \quad \text{if } X < \theta \tag{4.4}$$

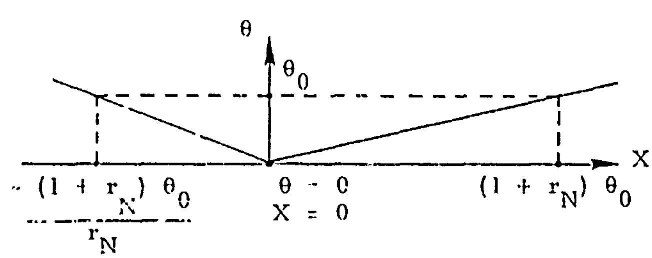
where

$$X = \frac{\delta M_F}{M_{FN}} + \frac{\delta r}{r_N} - \frac{\delta M_L}{M_{LN}} + \frac{\Delta r}{r_N} \quad . \tag{4.5}$$

The probability that $\theta < \theta_0$ is then the probability that X occurs along the strip

$$\frac{- (1 + r_N)}{r_N} \theta_0 < X < (1 + r_N) \theta_0 \quad . \tag{4.6}$$

Reference to the figure below will help clarify the above result.



Then

$$P(\theta < \theta_0) = \int_{x = \frac{-(1 + r_N)\theta_0}{r_N}}^{(1 + r_N)\theta_0} f(x)\, dx \qquad (4.7)$$

where f(x) is the distribution of x. Note that f(x) is normal with nonzero mean. That is,

$$f(x) = \frac{1}{\sqrt{2\pi}\, p}\, e^{-\frac{1}{2p^2}\left(x - \frac{\Delta r}{r_N}\right)^2} \qquad (4.8)$$

where

$$p^2 = \sigma_F^2 + \sigma_L^2 + \sigma_r^2 \quad . \qquad (4.9)$$

The above statistical determination was furnished by R. A. Moore. When Equation (4.8) is substituted into Equation (4.7) with a change in the integration variable, the result is

$$P(\theta < \theta_0) = \frac{1}{\sqrt{2\pi}} \int_{-\frac{(1 + r_N)\theta_0}{p\, r_N}}^{\frac{(1 + r_N)\theta_0}{p}} e^{-\frac{1}{2}\left(u - \frac{\Delta r}{p\, r_N}\right)^2} du \quad . \qquad (4.10)$$

Equation (4. 10) may be written in the form

$$P(\theta < \theta_0) = \frac{1}{\sqrt{2\pi}} \int_{-u_0}^{u_1} e^{-\frac{1}{2} u^2} du = \phi(u_0) + \phi(u_1) \tag{4. 11a}$$

where

$$\phi(u) = \frac{1}{\sqrt{2\pi}} \int_0^u e^{-\frac{1}{2} x^2} dx$$

$$u_1 = \frac{(1 + r_N) \theta_0}{p} - \frac{\Delta r}{p r_N} \tag{4. 11b}$$

$$u_0 = \frac{(1 + r_N) \theta_0}{p r_N} + \frac{\Delta r}{p r_N} \quad . \tag{4. 11c}$$

For example, if the mixture ratio $r_N = 1$, $\Delta r = 0$, $\theta_0 = \frac{1}{2} p$, then

$$P(\theta < \frac{1}{2} p) = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{-\frac{1}{2} u^2} du = 0.6826 \quad . \tag{4. 12}$$

That is, in this case the actual outage will be equal to or less than the rms outage about two-thirds of the time. This example is included only to show the inner consistency of the theory in an obvious case.

Equation (4.11) contains the parameter $\alpha = \frac{\Delta r}{p\, r_N}$ used in Section 3. The parameter α is still adjustable and may be chosen to maximize $P(\theta < \theta_0)$. This procedure will give a value for the parameter α that is different from the value obtained when the mean square outage was minimized. This procedure will then lead to a different loading mixture ratio. When the loading is based on maximizing $P(\theta < \theta_0)$, the mean square outage will be greater than with minimum mean square outage loading.

It is clear that the value of $\frac{\Delta r}{p\, r_N}$ that maximizes $P(\theta < \theta_0)$ is determined by equating Equations (4.11b) and (4.11c). Then

$$\frac{(1 + r_N)\theta_0}{p} - \frac{\Delta r}{p\, r_N} = \frac{(1 + r_N)\theta_0}{p\, r_N} + \frac{\Delta r}{p\, r_N}$$

or

$$\frac{\Delta r}{r_N} = \frac{r_N^{\,2} - 1}{2 r_N}\,\theta_0 \quad . \tag{4.13}$$

The value of α is obtained as

$$\alpha = \frac{r_N^{\,2} - 1}{2p\, r_N}\,\theta_0 \quad . \tag{4.14}$$

Substituting the calculated value of a into Equations (4. 11b) and (4. 11c) gives the result

$$u_0 = u_1 = \frac{(1 + r_N)^2 \, \theta_0}{2p \, r_N} \quad . \tag{4. 15}$$

Then, the maximum value of $P(\theta < \theta_0)$ is

$$P(\theta < \theta_0) = \frac{2}{\sqrt{2\pi}} \int_0^{u_0} e^{-\frac{1}{2} u^2} \, du = 2\phi(u_0) \tag{4. 16}$$

where u_0 is defined in Equation (4. 15).

The mean square outage that results when $P(\theta < \theta_0)$ is maximized may be calculated using Equations (3. 16) and (4. 14).

The probability that the outage is less than one percent, $P(\theta < 1\%)$, may be compared for each of the three methods of loading. As an example, consider the case where $r_N = 2.25$, $\sigma_F = 0.25\%$, $\sigma_L = 0.5\%$ and $\sigma_r = 0.5\%$. Then, as before,

$$\sigma = \sqrt{\sigma_F^2 + \sigma_L^2 + \sigma_r^2} = 0.75\% \quad . \tag{4. 17}$$

<u>Standard loading:</u>

By definition, $\Delta r = 0$, and from Equation (4. 11), $u_1 = 4.33$, $u_0 = 1.926$;

hence,

$$P(\theta < 1\%) = \frac{1}{\sqrt{2\pi}} \int_{-1.926}^{4.33} e^{-\frac{1}{2}u^2} \, du = 0.973 \quad . \tag{4.18}$$

Minimum mean square outage loading:

When $r_N = 2.25$, $\alpha = 0.60$; hence, from Equations (4.11), $u_1 = 3.74$ and $u_0 = 2.527$, and

$$P(\theta < 1\%) = 0.9941 \quad . \tag{4.19}$$

Maximum $P(\theta < 1\%)$ loading:

Substituting $r_N = 2.25$, $\theta_0 = 1\%$, $p = 0.75\%$ into Equation (4.15) gives $u_0 = 3.13$. Then, from Equation (4.16),

$$P(\theta < 1\%)^{*} = \frac{2}{\sqrt{2\pi}} \int_{0}^{3.13} e^{-\frac{1}{2}u^2} \, du = 0.9983 \quad . \tag{4.20}$$

The mean square outage that results when $P(\theta < 1\%)$ is minimized is easily calculated using Equation (3.16) (which is tabulated in Table V when $r_N = 2.25$). From Equation (4.14), $\alpha = 1.2$; hence, the resultant rms outage

---

*Similarly it may be shown that

$P(\theta < 0.402\%) = 0.80$	$P(\theta < 0.744\%) = 0.98$
$P(\theta < 0.526\%) = 0.90$	$P(\theta < 0.822\%) = 0.99$
$P(\theta < 0.626\%) = 0.95$	$P(\theta < 1.046\%) = 0.999$

is $\sqrt{\langle \theta^2 \rangle} = 0.375\%$. It will be noted from Equations (3.22) and (3.23) that the rms outage is increased over the value obtained for minimum mean square outage loading but less than that obtained with standard loading.

The loading mixture ratio is again computed using the result:

$$r_L = r_N \left(1 - \frac{\Delta r}{r_N}\right) \quad . \tag{4.21}$$

When $P(\theta < \theta_0)$ is maximized, $\frac{\Delta r}{r_N}$ is given by Equation (4.13), that is,

$$r_L = r_N - \frac{\theta_0}{2}\left(r_N^2 - 1\right) \quad , \tag{4.22}$$

SECTION 5

GENERAL DISTRIBUTION AND STATISTICS OF OUTAGE

In general, the outage frequency distribution, $f(\theta)$, may be obtained by differentiating Equation (4.11) as

$$f(\theta) = \left[\frac{\partial P(\theta < \theta_0)}{\partial \theta_0}\right]_{\theta_0 = \theta} \tag{5.1}$$

Hence,

$$f(\theta) = \frac{1}{\sqrt{2\pi}}\left[e^{-\frac{1}{2}u_1^2}\,\frac{(1+r_N)}{p} + e^{-\frac{1}{2}u_0^2}\,\frac{(1+r_N)}{p\,r_N}\right]$$

$$= \frac{1+r_N}{p\sqrt{2\pi}}\left\{e^{-\frac{1}{2p^2}\left[(1+r_N)\,\theta - \frac{\Delta r}{r_N}\right]^2} + \frac{1}{r_N}\,e^{-\frac{1}{2p^2 r_N^2}\left[(1+r_N)\,\theta + \Delta r\right]^2}\right\} \tag{5.2}$$

Mean outage:

When the distribution of outage is known, the mean outage, θ_N, is obtained as

$$\theta_N = \int_0^{\infty} \theta\, f(\theta)\, d\theta \tag{5.3}$$

When $f(\theta)$ from Equation (5.2) is substituted into Equation (5.3),

$$\theta_N = \frac{1 + r_N}{p\sqrt{2\pi}} \int_0^{\infty} \theta \left[e^{-\frac{1}{2p^2}\left\{(1 + r_N)\theta - \frac{\Delta r}{r_N}\right\}^2} + \frac{1}{r_N} e^{-\frac{1}{2p^2 r_N^2}\left\{(1 + r_N)\theta + \Delta r\right\}^2} \right] d\theta \tag{5.4}$$

The result of the integration is

$$\theta_N = p \left\{ \phi'(\alpha) + \alpha\,\phi(\alpha) - \frac{\alpha}{2}\,\frac{r_N - 1}{r_N + 1} \right\} \tag{5.5}$$

where

$$\alpha = \frac{\Delta r}{p\,r_N}$$

$$\phi(\alpha) = \frac{1}{\sqrt{2\pi}} \int_0^{\alpha} e^{-\frac{1}{2}u^2}\,du$$

$$\phi'(\alpha) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}\alpha^2} \tag{5.6}$$

A loading mixture ratio, r_L, may be obtained in such a way that θ_N is a minimum; then Δr will be a solution of the equation

$$\frac{\partial \theta_N}{\partial(\Delta r)} = 0 \tag{5.7}$$

The result of differentiating Equation (5.5) is obtained as

$$\phi''(a) + a\phi'(a) + \phi(a) = \frac{r_N - 1}{2(r_N + 1)} \tag{5.8}$$

where

$$\phi''(a) = -\frac{1}{\sqrt{2\pi}} a e^{-\frac{1}{2}a^2} \tag{5.9}$$

Since

$$\phi''(a) = -a\phi'(a) \tag{5.10}$$

it follows that

$$\phi(a) = \frac{1}{2}\frac{r_N - 1}{r_N + 1} \tag{5.11}$$

specifies the value of $a = \dfrac{\Delta r}{p r_N}$ that will minimize the mean outage, θ_N.

When the loading is chosen to minimize the mean outage, Equation (5.11) will apply. In this case, a simplification in Equation (5.5) is obtained. The result is

$$\theta_N = p\phi'(a) \tag{5.12}$$

Outage variance:

The outage variance (about the mean), σ_θ^2, is calculated as follows:

$$\sigma_\theta^2 = \left\langle (\theta - \theta_N)^2 \right\rangle$$

$$= \left\langle \theta^2 - 2\theta\, \theta_N + \theta_N^2 \right\rangle$$

$$= \left\langle \theta^2 \right\rangle - 2\theta_N^2 + \theta_N^2$$

$$= \left\langle \theta^2 \right\rangle - \theta_N^2 \tag{5.13}$$

The definition

$$\theta_N = < \theta > \tag{5.14}$$

was used in the above derivation.

The quantity $<\theta^2>$ is the mean square outage derived in Section 3. This same expression may be alternately obtained as

$$< \theta^2 > = \int_0^\infty \theta^2\, f(\theta)\, d\theta \tag{5.15}$$

When the outage frequency distribution, $f(\theta)$, from Equation (5.2) is substituted into Equation (5.15) and the integration is performed, the result is

$$< \theta^2 > = p^2 \frac{r_N^2 + 1}{2(1 + r_N)^2} (1 + a^2)$$

$$- \frac{r_N - 1}{r_N + 1} p^2 \left[a \phi'(a) + (1 + a^2) \phi(a) \right] \qquad (5.16)$$

Note that Equation (5.16) is in agreement with Equation (3.16). The expression for mean square outage is simplified when the missile is loaded so as to minimize the mean outage. In this case

$$< \theta^2 > = p^2 \left[\frac{(1 + a^2) r_N}{(1 + r_N)^2} - \frac{r_N - 1}{r_N + 1} a \phi'(a) \right] \qquad (5.17)$$

The outage variance when the missile is loaded to minimize the mean outage is

$$\sigma_\theta^2 = p^2 \left\{ \frac{(1 + a^2) r_N}{(1 + r_N)^2} - \frac{r_N - 1}{r_N + 1} a \phi'(a) - \left[\phi'(a) \right]^2 \right\} \qquad (5.18)$$

When the missile is loaded in any other way (i.e., such that Equation (5.12) is not satisfied), then σ_θ^2 may be obtained using Equation (5.13). That is,

$$\sigma_\theta^2 = < \theta^2 > - \theta_N^2$$

where $< \theta^2 >$ is given by Equation (5.16) and θ_N is given by Equation (5.5).

Probability that the outage is fuel or oxidizer

The probability that the outage is either fuel or oxidizer may be determined from Equation (4.4). When $X \geq 0$, the outage is fuel; when $X < 0$, the outage is oxidizer. Now X is a random variable with mean,

$$\langle X \rangle = \frac{\Delta r}{r_N}$$

The distribution of X is given in Equation (4.8). The probability that the outage is fuel is equal to the probability that $X \geq 0$. Then:

$$P(X \geq 0) = \frac{1}{\sqrt{2\pi}\, p} \int_0^{\infty} e^{-\frac{1}{2p^2}\left(x - \frac{\Delta r}{r_N}\right)^2} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{u = -a}^{\infty} e^{-\frac{1}{2}u^2} du$$

$$= \frac{1}{2} + \phi(a) \qquad (5.19)$$

where $\phi(a)$ is defined in Equation (5.6). The outage can only be fuel or oxidizer. That is,

$$P(X \geq 0) + P(X < 0) = 1$$

or

$$P(X < 0) = \frac{1}{2} - \phi(a) \quad . \tag{5.20}$$

Hence,

$$P(\theta = \text{fuel}) = \frac{1}{2} + \phi(a)$$

$$P(\theta = \text{oxidizer}) = \frac{1}{2} - \phi(a) \quad . \tag{5.21}$$

Note that when the ratio of the two probabilities is equal to r_N,

$$\frac{\frac{1}{2} + \phi(a)}{\frac{1}{2} - \phi(a)} = r_N$$

or

$$\phi(a) = \frac{1}{2} \frac{r_N - 1}{r_N + 1} \quad . \tag{5.22}$$

Equation (5.22) is identical with Equation (5.11), hence the above assumptions pertain to minimum mean outage loading.

SECTION 6

BIASING TO MAXIMIZE MISSILE PERFORMANCE

A particular missile configuration is usually required to place a payload into a ballistic orbit. A single parameter, V, may be examined to determine whether or not the given missile configuration will have the capability of achieving the mission. The discussion in this section will assume that V refers to the burnout velocity magnitude. No loss in generality is introduced by this assumption because V could equally well refer to the burnout energy, the total range at earth impact, or any other quantity with the desired property.

The nominal velocity of the missile at burnout, V_N, is a function of many variables, for example

$$V_N = V_N (M_N, F_N, I_N, \cdots, \theta_N) \tag{6.1}$$

where

M_N = expected liftoff weight

F_N = expected thrust

I_N = expected specific impulse

θ_N = expected mean outage

The three dots in expression (6.1) refer to other variations not specifically listed. These include trajectory shaping, missile steering with guidance, payload weight, etc.

Expression (6.1) may be expanded in a Taylor's series to obtain

$$V = V_N + A\,\Delta M + B\,\Delta F + C\,\Delta I + \cdots - D\,\Delta\theta \tag{6.2}$$

where

ΔM = liftoff weight variation

ΔF = thrust variation

ΔI = specific impulse variation

$\Delta\theta = \theta - \theta_N$ = outage variation

and A, B, C, $\cdots$, D are the corresponding partial derivatives. That is,

$$A = \frac{\partial V}{\partial M}, \quad B = \frac{\partial V}{\partial F}, \quad C = \frac{\partial V}{\partial I}, \quad \cdots, \quad D = -\frac{\partial V}{\partial \theta} \tag{6.3}$$

where the partial derivatives are evaluated at burnout. Note that the sign on D is chosen in expression (6.3) such that $D > 0$. It is assumed that ΔM, ΔF, ΔI, $\cdots$, $\Delta\theta$ have zero mean but not necessarily that these variations are normally distributed. Furthermore, it is assumed that M_N, F_N, I_N + $\cdots$ are specified by the missile configuration. This is not the case for θ_N. The value for θ_N depends on the missile loading procedure (or how the PU system is biased).

Equation (6.2) may be simplified by defining a nominal final velocity, V_N^*, that does not depend on θ_N, as

$$V_N^* = V_N + D\theta_N \tag{6.4}$$

The quantity V_N^* is obtained by using the nominal expected values for M, F, I, $\cdots$, and allowing the missile to burn out with zero outage. Then

$$V = V_N^* - D\,\theta_N + A\,\Delta M + B\,\Delta F + C\,\Delta I + \cdots - D(\theta - \theta_N) \qquad (6.5)$$

It is clear that if the performance of the nominal missile is to be maximized, the missile should be loaded to maximize θ_N. The loading mixture ratio, $r_L = r_N - \Delta r$, that will maximize θ_N is obtained in Equation (5.11).

It may be desired to maximize the probability that the missile performance (as measured by the parameter V) is greater then some fixed value, V_0. The development in the following paragraphs applies to this case.

It is desired that the missile achieve a required velocity, V_0, with a high probability. The probability that V will be greater then V_0, $P(V > V_0)$, may then be written as

$$P(V > V_0) = \int_{V = V_0}^{\infty} f(V)\, dV \qquad (6.6)$$

where f (V) is the frequency distribution of V. It is assumed there are sufficient random variables in Equation (6.5) to justify the use of the central limit theorem. Then the distribution of V will be normal with mean

$$\langle V \rangle = V_N = V_N^* - D\,\theta_N \qquad (6.7)$$

and variance

$$\sigma_V^2 = \left\langle \left[A\,\Delta M + B\,\Delta F + C\,\Delta I + \cdots - D\,(\theta - \theta_N) \right]^2 \right\rangle$$

$$\cong A^2 \sigma_M^2 + B^2 \sigma_F^2 + C^2 \sigma_I^2 + \cdots + D^2 \sigma_\theta^2$$

$$= A^2 \sigma_M^2 + B^2 \sigma_F^2 + C^2 \sigma_I^2 + \cdots + D^2 \sigma_\theta^2 \tag{6.8}$$

where σ_M, σ_F, σ_I, $\cdots$, σ_θ denote the one-sigma variations about the expected mean of the variables M, F, I, $\cdots$, θ. Note that all cross correlations were assumed to vanish. Hence, Equation (6.6) may be written as

$$P(V > V_0) = \frac{1}{\sqrt{2\pi}\,\sigma_V} \int_{V = V_0}^{\infty} e^{-\frac{1}{2}\frac{(V - V_N)^2}{\sigma_V^2}} \tag{6.9}$$

When the substitution

$$u - \frac{V - V_N}{\sigma_V} \tag{6.10}$$

is made in Equation (6.9), the following result is obtained

$$P(V > V_0) = \frac{1}{\sqrt{2\pi}} \int_{u = u_0}^{u = \infty} e^{-\frac{1}{2}u^2}\, du \tag{6.11}$$

where

$$u_0 = \frac{V_0 - V_N}{\sigma_V} = \frac{V_0 - V_N^* + D\theta_N}{\sigma_V} \tag{6.12}$$

It is desired that the missile stage be loaded so that $P(V > V_0)$ is a maximum. It is clear from Equation (6. 11) that this is equivalent to biasing such that u_0 is a minimum. When $\frac{\partial u_0}{\partial \Delta r}$ is formed and set equal to zero, the result is:

$$\frac{\partial \theta_N}{\partial \Delta r} = \frac{V_0 - V_N^* + D\theta_N}{D\sigma_V} \frac{\partial \sigma_V}{\partial \Delta r} \tag{6.13}$$

When Δr is obtained as a solution of Equation (6. 13), the loading mixture ratio $r_L = r_N - \Delta r$ is obtained that will maximize $P(V > V_0)$.

Equation (6. 13) may be written in several alternate forms. When it is assumed that ΔM, ΔF, ΔI, etc., are independent of Δr, the following result is obtained:

$$\sigma_V \frac{\partial \sigma_V}{\partial \Delta r} = \frac{1}{2} \frac{\partial \sigma_\theta^2}{\partial \Delta r} \tag{6.14}$$

Hence, Equation (6. 13) may be written as

$$\frac{\partial \theta_N}{\partial \Delta r} = \frac{V_0 - V_N^* + D\theta_N}{2\sigma_V^2(0)} \frac{\partial \sigma_\theta^2}{\partial \Delta r} \tag{6.15}$$

Now

$$\sigma_\theta^2 = \langle \theta^2 \rangle - \theta_N^2 \quad . \tag{6.16}$$

Hence, Equation (6.15) becomes

$$\frac{\partial \theta_N}{\partial \Delta r} \left(1 + 2\lambda\, \theta_N\right) = \lambda \frac{\partial \langle \theta^2 \rangle}{\partial \Delta r} \tag{6.17}$$

where

$$\lambda = \frac{V_0 - V_N^* + \dot{D}\theta_N}{2\sigma_V^2(\theta)} \quad . \tag{6.18}$$

Equation (6.17) is best solved for Δr by iterative methods. The resulting value of Δr will specify the loading procedure that maximizes $P(V > V_0)$. When Δr is known, the quantities θ_N, $\langle \theta^2 \rangle$, and σ_θ^2 may be calculated. Then $\sigma_V^2(\theta)$ and $P(V > V_0)$ may be evaluated.

SECTION 7

GENERAL DISCUSSION OF A PROPELLANT UTILIZATION SYSTEM

Outage is minimized in the Thor and Titan missiles using an open-loop system. This method requires that close tolerances be maintained in propellant loading and the burning mixture ratio. The purpose of a PU system (as used in the Atlas) is to cause the fuel and oxidizer to be expended simultaneously using closed-loop measurement and control.

The operation of a PU system requires that the remaining amounts of fuel and oxidizer, F and L, respectively, be measured. Then

$$F^{*} = F - f \tag{7.1}$$

$$L^{*} = L - \ell \tag{7.2}$$

where

F^{*}, L^{*} = remaining fuel and oxidizer measured

F, L = actual fuel and oxidizer remaining

f, ℓ = errors in the fuel and oxidizer measurements

The measurement errors f and ℓ are assumed in this analysis to be constants for any one flight. The units are mass (pounds) of fuel or oxidizer, respectively. The frequency distribution of these errors for an ensemble of flights is assumed to be gaussian with mean zero. The measured tank mixture ratio may then be defined as

$$R^{*} = \frac{L^{*}}{F^{*}} \tag{7.3}$$

If the actual burning mixture ratio, R, is controlled to equal R^*, then the equation of operation of the PU system may be written as

$$R = R^* + \delta r \tag{7.4}$$

where

$$R = \frac{\dot{L}}{\dot{F}}$$

δr = gaussian constant error in controlling the mixture ratio

$\dot{L}$ = actual oxidizer flow rate

$\dot{F}$ = actual fuel flow rate

Equation (7.4) may be written as

$$\frac{\dot{L}}{\dot{F}} = \frac{L - \ell}{F - f} + \delta r \quad . \tag{7.5}$$

It may now be assumed, without loss in generality, that

$$\dot{F} = \dot{F}_N, \text{ a constant} \tag{7.6}$$

or

$$F = F_0 + \dot{F}_N t \tag{7.7}$$

where

F_0 = actual initial mass of fuel loaded.

Equation (7.5) may then be integrated to obtain the functions $L(t)$ and $R(t)$. The result of this integration shows that $R(t)$ varies as a function of loading, measurement, and control errors.

The above unpredictable variation in $R(t)$ can be avoided by controlling the burning mixture ratio, $R(t)$, to the nominal burning mixture ratio, r_N. In this system, the flow rate of the oxidizer or the fuel is controlled to be proportional to the difference between the measured tank mixture ratio and the nominal burning mixture. The operation of this system is described by the equation

$$\dot{L} - \dot{L}_N = K\,(R^* + \delta r - r_N) \tag{7.8}$$

where

K = constant of proportionality

$\dot{L}_N$ = nominal oxidizer flow rate

r_N = nominal burning mixture ratio = $\dfrac{\dot{L}_N}{\dot{F}_N}$

Note that Equation (7.8) is written assuming that the oxidizer flow rate is adjusted. The fuel flow rate may be chosen to be nominal without loss in generality. Note also that if $K = -\dot{F}_N$, the system described by Equation (7.5) results. Since this system may be shown to have some undesirable characteristics, the constant K will be chosen such that

$$K \neq \dot{F}_N \quad . \tag{7.9}$$

In general, K may be written as

$$K = \mu \dot{F}_N, \quad \mu \neq 1 \tag{7.10}$$

where μ may be referred to as the PU system gain.

The system operation described by Equation (7.8) closely approximates the Convair PU system used on the Atlas missile. Equation (7.8) may be written as

$$\dot{L} - \dot{L}_N = \mu \dot{F}_N \left(\frac{L - \ell}{F - f} + \delta r - r_N \right) \tag{7.11}$$

or

$$\dot{L} - \frac{\mu \dot{F}_N}{F - f} L = \dot{L}_N - \mu \dot{F}_N \left(r_N - \delta r + \frac{\ell}{F - f} \right) . \tag{7.12}$$

Equation (7.12) may be integrated to obtain L(t) when

$$F = F_0 + \dot{F}_N t \tag{7.13}$$

and $\dot{F}_N$ is a constant. The result of this integration, when $\mu \neq 1$, is given below.

$$L - \ell = \left(r_N - \frac{\mu}{\mu - 1} \delta r \right) (F - f) + \epsilon \tag{7.14}$$

where

$$\epsilon = \left(\frac{F - f}{F_0 - f}\right)^{\mu} \left[L_0 - \ell - \left(r_N - \frac{\mu}{\mu - 1}\,\delta r\right)\left(F_0 - f\right)\right].$$

L_0 = actual initial mass of oxidizer loaded. (7.15)

Equations (7.13) and (7.14) describe the PU system operation under the assumption that the fuel flow rate is a constant. Now consider that the PU system has been operating for some time. Then

$$F << F_0 \qquad (7.16)$$

and

$$L << L_0 \quad . \qquad (7.17)$$

Hence, if the PU system gain, μ, is large, then ϵ will be small compared with the first term in Equation (7.14). The magnitude of ϵ is further reduced if an attempt is made to load the propellant in accordance with the nominal burning mixture ratio r_N. Then, since ϵ is a second-order term,

$$L - \ell = \left(r_N - \frac{\mu}{\mu - 1}\,\delta r\right)\left(F - f\right) \quad . \qquad (7.18)$$

Satisfactory operation of the PU system will be obtained for any large value of μ. For example, μ may be chosen in the range

$$2 < \mu < 5 \quad .$$

Equation (7.18) was derived from Equation (7.11), which requires the oxidizer flow rate to change linearly with the error signal

$$\mu \dot{F}_N \left(\frac{L - \ell}{F - f} - r_N \right) \qquad (7.19)$$

In any practical system there is a limit to the allowable change in the oxidizer flow rate. Hence, in the initial-operation phase of the PU system, the oxidizer control valve may be hard over (one way or the other) and Equation (7.11) will not apply. It is assumed, however, that the tank mixture ratio will eventually be controlled so that Equation (7.11) does apply. If the PU system gain is not too high, the system should then continue to be linear. If μ is too high, an undesirable limit cycle may result because of inertia in the valve hardware.

The quantities F_0 and L_0 do not appear in Equation (7.18). It then may be concluded that outage is not a function of initial loading errors. As the propellant approaches depletion, the mixture ratio control error, δr, becomes a second-order error, and (independent of the PU system gain) Equation (7.18) reduces to

$$L - \ell = r_N (F - f) \quad . \qquad (7.20)$$

Note that when F and L have been sufficiently reduced, Equation (7.20) requires that

$$R \equiv \frac{\dot{L}}{\dot{F}} = r_N \quad .$$

Because of the fast reaction time of the system (with high μ), this equation will be true even when r_N and $\dot{F}$ have small variations with time.

The propellant measurement system may be designed such that L^* and F^* become ambiguous when only a small amount of propellant remains in the tanks. If this is the case, it will be assumed that the system goes open loop prior to the occurrence of the ambiguous measurements with $R = r_N$ until one of the propellants is exhausted. A small control error during this process may be neglected because only a small amount of propellant remains. When this procedure is followed, no change in Equation (7.20) is required.

The outage, whether it is oxidizer or fuel, may now be determined from Equation (7.20) as a function of the measurement errors ℓ and f which were assumed to be constant in this equation. Because of the fast reaction time of the system, f and ℓ may be time-varying quantities. The values of f and ℓ to be used in calculating outage are the values of f and ℓ that persist when the propellant is nearly expended.

SECTION 8

OPERATION OF A PROPELLANT UTILIZATION SYSTEM

Equation (7.20) may be used to compute the outage that results when no biasing technique is employed. When the fuel, F, is exhausted, the outage is oxidizer and

$$\theta_L = \ell - r_N f \geq 0 \qquad (8.1a)$$

When L = 0, Equation (7.20) may be solved for the fuel outage, θ_F, with the result

$$\theta_F = f - \frac{\ell}{r_N} \geq 0 \qquad (8.1b)$$

Equations (8.1a) and (8.1b) may now be compared with Equations (2.9a) and (2.9b). The mean square outage may again be calculated using Equations (8.1a) or (8.1b) whichever gives a positive result. It will be assumed that f and ℓ are uncorrelated and normally distributed with mean zero. Then the outage is equally likely to be all fuel or all oxidizer. The mean square outage, $< \theta^2 >$, written as a fraction of the total propellant, may then be evaluated as

$$< \theta^2 > = \frac{Z_f^2 + Z_\ell^2}{2} \qquad (8.2)$$

where Z_f^2 and Z_ℓ^2 are mean square fuel and oxidizer outages expressed as fractions of the total propellant. These quantities may be calculated from

Equations (8.1) by squaring, averaging, and dividing by the square of the nominal mass of propellant, as:

$$Z_f^2 = \frac{\left\langle \theta_F^2 \right\rangle}{M_N^2}$$

$$= \frac{1}{(1 + r_N)^2} \left(\sigma_f^2 + \sigma_\ell^2\right) \tag{8.3}$$

and

$$Z_\ell^2 = \frac{\left\langle \theta_L^2 \right\rangle}{M_N^2}$$

$$= \frac{\left\langle r_N^2 \right\rangle}{(1 + r_N)^2} \left(\sigma_f^2 + \sigma_\ell^2\right) \tag{8.4}$$

where

$$\sigma_f^2 = \left\langle \left| \frac{f}{M_{FN}} \right|^2 \right\rangle \tag{8.5}$$

$$\sigma_\ell^2 = \left\langle \left| \frac{\ell}{M_{LN}} \right|^2 \right\rangle \tag{8.6}$$

and M_N, M_{FN}, and M_{LN} again denote the nominal total mass of propellant, fuel, and oxidizer loaded. The symbol $< \cdots >$ indicates an ensemble average over a large number of flights. When the above results are substituted into Equation (8.2), the expression for mean square outage is obtained as

$$< \theta^2 > = \frac{r_N^2 + 1}{2(1 + r_N)^2} \, q^2 \tag{8.7}$$

where

$$q^2 = \sigma_f^2 + \sigma_\ell^2$$

σ_f = rms fuel measurement error (1σ), expressed as a fraction of M_{FN}

σ_ℓ = rms oxidizer measurement error (1σ), expressed as a fraction of M_{LN} (8.8)

The expression for mean square outage given by Equation (8.7) assumes no biasing of the PU system. That is the system operation is specified by Equation (7.11).

The similarity between Equations (8.7) and (2.14) should be noted. The equations are identical except that p has been replaced by q.

The operation of a PU system can be improved by introducing a bias into the measurements of either the fuel or the oxidizer. When such a bias, Δf, is introduced in the fuel measurement, the oxidizer or fuel outage expressed as fractions of the tot. ' propellant becomes

$$\frac{L}{M_N} = \frac{r_N}{1 + r_N} \left(\frac{f}{M_{FN}} - \frac{\ell}{M_{LN}} + \frac{\Delta f}{M_{FN}} \right) \tag{8.9a}$$

or

$$\frac{F}{M_N} = \frac{1}{1 + r_N} \left(\frac{f}{M_{FN}} - \frac{\ell}{M_{LN}} + \frac{\Delta f}{M_{FN}} \right) \tag{8.9b}$$

These equations should be compared with Equations (4.1) and (4.2). An analysis nearly identical with that used in Section 4 will derive the expression for $P(\theta < \theta_0)$ when a PU system is employed. The result of this derivation is

$$P(\theta < \theta_0) = \frac{1}{\sqrt{2\pi}} \int_{V = V_0}^{V = V_1} e^{-\frac{1}{2} V^2} \, dV \tag{8.10a}$$

where

$$V_1 = \frac{(1 + r_N)\,\theta_0}{q} - \frac{\Delta f}{q M_{FN}} \tag{8.10b}$$

$$V_0 = \frac{(1 + r_N)\,\theta_0}{q\, r_N} + \frac{\Delta f}{q M_{FN}} \tag{8.10c}$$

Equations (8.10) should be compared with Equation (4.11) and the similarity noted. These equations will be the same if the following identification is made:

$$p \equiv q \tag{8.11a}$$

$$\alpha = \frac{\Delta r}{p r_N} \equiv \frac{\Delta f}{q M_{FN}} \tag{8.11b}$$

The above identification may be used in Equations (3.16), (4.11), (4.16), (5.5), (5.17), etc. Then all of the relations (derived on the assumption that conventional loading techniques would be employed) may be extended to apply to PU system operation. Further, it should be observed that the PU system fuel measurement bias, Δf, is equivalent to the fuel loading bias, ΔM_F, discussed in Equation (3.7a).

Each of the loading methods described in Sections 3, 4, and 5 result in a value for the parameter $\alpha = \frac{\Delta r}{p r_N}$. When α and $p(= q)$ are known, Δr or ΔM_F (= Δf) may be determined using Equation (8.11b).

Proper operation of the propellant utilization system requires that the quantity Δf be subtracted from the fuel measurement, F^*. The oxidizer flow rate is then controlled according to the equation

$$\dot{L} - \dot{L}_N = \mu \dot{F}_N \left(\frac{L^*}{F^* - \Delta f} + \delta r - r_N \right) \tag{8.12}$$

rather than Equation (7.11).

An oxidizer measurement bias $\Delta \ell$, could have been determined rather than Δf. This quantity would be obtained from Δf as

$$\frac{\Delta \ell}{M_{LN}} = - \frac{\Delta f}{M_{FN}} \tag{8.13}$$

In this case the oxidizer flow rate should be controlled by the equation

$$\dot{L} - \dot{L}_N = \mu \dot{F}_N \left(\frac{L^* - \Delta \ell}{F^*} + \delta r - r_N \right) \tag{8.14}$$

rather than Equation (7.11).

UNCLASSIFIED

Aerospace Corporation, El Segundo, California.
OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe. 1 May 1961. 66p. incl tables (Report No. TDR-594-(1114) TN-2) (Contract AF 04(647)-594)

Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $<\theta^2>$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California.
OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe, 1 May 1961, 66p. incl tables (Report No. TDR-594-(1114) TN-2) (Contract AF 04(647)-594)

Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $<\theta^2>$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California.
OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe. 1 May 1961. 66p. incl. tables (Report No. TDR-594-(1114) TN-2) (Contract AF 04(647)-594)

Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $<\theta^2>$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California.
OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe. 1 May 1961. 66p. incl. tables (Report No. TDR-594-(1114) TN-2) (Contract AF 04(647)-594)

Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $<\theta^2>$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California. OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe, 1 May 1961. 66p. incl. tables (Report No. TDR-594-(1114)TN-2) (Contract AF 04(647)-594) Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $\langle\theta^2\rangle$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California. OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe, 1 May 1961. 66p. incl. tables (Report No. TDR-594-(1114)TN-2) (Contract AF 04(647)-594) Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $\langle\theta^2\rangle$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California. OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe, 1 May 1961. 66p. incl. tables (Report No. TDR-594-(1114) TN-2) (Contract AF 04(647)-594) Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $\langle\theta^2\rangle$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California. OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe, 1 May 1961. 66p. incl. tables (Report No. TDR-594-(1114) TN-2) (Contract AF 04(647)-594) Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $\langle\theta^2\rangle$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California. OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe. 1 May 1961. 66p. incl. tables (Report No. TDR-594-(1114)TN-2) (Contract AF 04(647)-594)

Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $\langle\theta^2\rangle$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California. OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe. 1 May 1961. 66p. incl. tables (Report No. TDR-594-(1114)TN-2) (Contract AF 04(647)-594)

Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $\langle\theta^2\rangle$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California. OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe. 1 May 1961. 66p. incl. tables (Report No. TDR-594-(1114)TN-2) (Contract AF 04(647)-594)

Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $\langle\theta^2\rangle$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California. OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe. 1 May 1961. 66p. incl. tables (Report No. TDR-594-(1114) TN-2) (Contract AF 04(647)-594)

Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $\langle\theta^2\rangle$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California. OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe. 1 May 1961. 66p. incl. tables (Report No. TDR-594-(1114)TN-2) (Contract AF 04(647)-594)

Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $<\theta^2>$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California. OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe. 1 May 1961. 66p. incl. tables (Report No. TDR-594-(1114)TN-2) (Contract AF 04(647)-594)

Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $<\theta^2>$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California. OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe. 1 May 1961. 66p. incl. tables (Report No. TDR-594-(1114) TN-2) (Contract AF 04(647)-594)

Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $<\theta^2>$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED

UNCLASSIFIED

Aerospace Corporation, El Segundo, California. OPTIMUM PROPELLANT LOADING AND PROPELLANT UTILIZATION SYSTEM TECHNIQUES (A TUTORIAL REPORT), by David W. Whitcombe. 1 May 1961. 66p. incl. tables (Report No. TDR-594-(1114) TN-2) (Contract AF 04(647)-594)

Unclassified Report

This report derives optimum fuel or mixture ratio biasing techniques commonly employed in current ballistic missile programs. The missile stages can use conventional loading (e.g., Titan and Thor) or they can use propellant utilization (PU) systems (e.g., Atlas and Centaur). A description of the mechanization errors leading to unburned propellant (outage) is given for both cases. Formulae, derived in the report, allow the calculation of mean outage, θ_N; the mean square outage, $<\theta^2>$; the outage variance, σ_θ^2; as well as the probability that the outage is less than some fixed value, $P(\theta<\theta_0)$.

(over)

UNCLASSIFIED